

## 9MA0/02: Pure Mathematics Paper 2 Mark scheme

Question	Scheme	Marks	AOs
<b>1</b>	$\frac{1}{2}r^2(4.8)$	M1	1.1a
	$\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e.	A1	1.1b
	length of minor arc = $7.5(2\pi - 4.8)$	dM1	3.1a
	$= 15\pi - 36$ $\{a = 15, b = -36\}$	A1	1.1b
		<b>(4)</b>	
<b>1 Alt</b>	$\frac{1}{2}r^2(4.8)$	M1	1.1a
	$\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e.	A1	1.1b
	length of major arc = $7.5(4.8)$ $\{= 36\}$		
	length of minor arc = $2\pi(7.5) - 36$	dM1	3.1a
	$= 15\pi - 36$ $\{a = 15, b = -36\}$	A1	1.1b
		<b>(4)</b>	
<b>(4 marks)</b>			
<b>Question 1 Notes:</b>			
<b>M1:</b>	Applies formula for the area of a sector with $\theta = 4.8$ ; i.e. $\frac{1}{2}r^2\theta$ with $\theta = 4.8$ <b>Note:</b> Allow M1 for considering ratios. E.g. $\frac{135}{\pi r^2} = \frac{4.8}{2\pi}$		
<b>A1:</b>	Uses a correct equation (e.g. $\frac{1}{2}r^2(4.8) = 135$ ) to obtain a radius of 7.5		
<b>dM1:</b>	Depends on the previous M mark. A complete process for finding the length of the minor arc $AB$ , by either <ul style="list-style-type: none"> <li>• <math>(\text{their } r) \times (2\pi - 4.8)</math></li> <li>• <math>2\pi(\text{their } r) - (\text{their } r)(4.8)</math></li> </ul>		
<b>A1:</b>	Correct exact answer in its simplest form, e.g. $15\pi - 36$ or $-36 + 15\pi$		

Question	Scheme	Marks	AOs
<b>2(a)</b>	Attempts to substitute $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ into either $1 + 4\cos \theta$ or $3\cos^2 \theta$	M1	1.1b
	$1 + 4\cos \theta + 3\cos^2 \theta \approx 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \frac{1}{2}\theta^2\right)^2$		
	$= 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$	M1	1.1b
	$= 1 + 4 - 2\theta^2 + 3 - 3\theta^2 + \frac{3}{4}\theta^4$		
	$= 8 - 5\theta^2$ *	A1*	2.1
		<b>(3)</b>	
<b>(b)(i)</b>	E.g. <ul style="list-style-type: none"> <li>Adele is working in degrees and not radians</li> <li>Adele should substitute <math>\theta = \frac{5\pi}{180}</math> and not <math>\theta = 5</math> into the approximation</li> </ul>	B1	2.3
<b>(b)(ii)</b>	$8 - 5\left(\frac{5\pi}{180}\right)^2 = \text{awrt } 7.962$ , so $\theta = 5^\circ$ gives a good approximation.	B1	2.4
		<b>(2)</b>	
<b>(5 marks)</b>			

**Question 2 Notes:**

<b>(a)(i)</b>	
<b>M1:</b>	See scheme
<b>M1:</b>	Substitutes $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ into $1 + 4\cos \theta + 3\cos^2 \theta$ and attempts to apply $\left(1 - \frac{1}{2}\theta^2\right)^2$
	<b>Note:</b> It is not a requirement for this mark to write or refer to the term in $\theta^4$
<b>A1*:</b>	Correct proof with no errors seen in working.
	<b>Note:</b> It is not a requirement for this mark to write or refer to the term in $\theta^4$
<b>(a)(ii)</b>	
<b>B1:</b>	See scheme
<b>(b)(i)</b>	
<b>B1:</b>	See scheme
<b>(b)(ii)</b>	
<b>B1:</b>	Substitutes $\theta = \frac{5\pi}{180}$ or $\frac{\pi}{36}$ into $8 - 5\theta^2$ to give awrt 7.962 <b>and</b> an appropriate conclusion.

Question	Scheme	Marks	AOs
<b>3 (a)</b>	$\{t = 0, \theta = 75 \Rightarrow 75 = 25 + A \Rightarrow A = 50\} \Rightarrow \theta = 25 + 50e^{-0.03t}$	B1	3.3
		(1)	
<b>(b)</b>	$\{\theta = 60 \Rightarrow \} \Rightarrow 60 = 25 + "50"e^{-0.03t} \Rightarrow e^{-0.03t} = \frac{60 - 25}{"50"}$	M1	3.4
	$t = \frac{\ln(0.7)}{-0.03} = 11.8891648 = 11.9 \text{ minutes (1 dp)}$	A1	1.1b
		(2)	
<b>(c)</b>	A valid evaluation of the model, which relates to the large values of $t$ . E.g. <ul style="list-style-type: none"> <li>As <math>20.3 &lt; 25</math> then the model is not true for large values of <math>t</math></li> <li><math>e^{-0.03t} = \frac{20.3 - 25}{"50"} = -0.094</math> does not have any solutions and so the model predicts that tea in the room will never be <math>20.3^\circ\text{C}</math>. So the model does not work for large values of <math>t</math></li> <li><math>t = 120 \Rightarrow \theta = 25 + 50e^{-0.03(20)} = 25.12\dots</math> which is not approximately equal to <math>20.3</math>, so the model is not true for large values of <math>t</math></li> </ul>	B1	3.5a
		(1)	
<b>(4 marks)</b>			
<b>Question 3 Notes:</b>			
<b>(a)</b>			
<b>B1:</b>	Applies $t = 0, \theta = 75$ to give the complete model $\theta = 25 + 50e^{-0.03t}$		
<b>(b)</b>			
<b>M1:</b>	Applies $\theta = 60$ and their value of $A$ to the model and rearranges to make $e^{-0.03t}$ the subject.		
	<b>Note:</b> Later working can imply this mark.		
<b>A1</b>	Obtains 11.9 (minutes) with no errors in manipulation seen.		
<b>(c)</b>			
<b>B1</b>	See scheme		

Question	Scheme	Marks	AOs	
<b>4(a)</b>		Correct graph in quadrant 1 and quadrant 2 with V on the x-axis	B1	1.1b
	States $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$ or $\frac{5}{2}$ marked in the correct position on the x-axis and 5 marked in the correct position on the y-axis	B1	1.1b	
		<b>(2)</b>		
<b>(b)</b>	$ 2x - 5  > 7$			
	$2x - 5 = 7 \Rightarrow x = \dots$ and $-(2x - 5) = 7 \Rightarrow x = \dots$	M1	1.1b	
	{critical values are $x = 6, -1 \Rightarrow$ } $x < -1$ or $x > 6$	A1	1.1b	
		<b>(2)</b>		
<b>(c)</b>	$ 2x - 5  > x - \frac{5}{2}$			
	<p>E.g.</p> <ul style="list-style-type: none"> <li>Solves <math>2x - 5 = x - \frac{5}{2}</math> to give <math>x = \frac{5}{2}</math> and solves <math>-(2x - 5) = x - \frac{5}{2}</math> to also give <math>x = \frac{5}{2}</math></li> <li>Sketches graphs of <math>y =  2x - 5 </math> and <math>y = x - \frac{5}{2}</math>. Indicates that these graphs meet at the point <math>\left(\frac{5}{2}, 0\right)</math></li> </ul>	M1	3.1a	
	<p>Hence using set notation, e.g.</p> <ul style="list-style-type: none"> <li><math>\left\{x: x &lt; \frac{5}{2}\right\} \cup \left\{x: x &gt; \frac{5}{2}\right\}</math></li> <li><math>\left\{x \in \mathbb{R}, x \neq \frac{5}{2}\right\}</math></li> <li><math>\mathbb{R} - \left\{\frac{5}{2}\right\}</math></li> </ul>	A1	2.5	
		<b>(2)</b>		
			<b>(6 marks)</b>	

Question 4 Notes:	
(a)	
<b>B1:</b>	See scheme
<b>B1:</b>	See scheme
(b)	
<b>M1:</b>	See scheme
<b>A1:</b>	Correct answer, e.g. <ul style="list-style-type: none"> <li>• <math>x &lt; -1</math> or <math>x &gt; 6</math></li> <li>• <math>x &lt; -1 \cup x &gt; 6</math></li> <li>• <math>\{x: x &lt; -1\} \cup \{x: x &gt; 6\}</math></li> </ul>
(c)	
<b>M1:</b>	A complete process of finding that $y =  2x - 5 $ and $y = x - \frac{5}{2}$ meet at <i>only</i> one point. This can be achieved either algebraically or graphically.
<b>A1:</b>	See scheme. <b>Note:</b> Final answer must be expressed using set notation.

Question	Scheme	Marks	AOs
<b>5</b>	$3x - 2y = k$ intersects $y = 2x^2 - 5$ at two distinct points		
	Eliminate $y$ and forms quadratic equation $= 0$ or quadratic expression $\{= 0\}$	M1	3.1a
	$\{3x - 2(2x^2 - 5) = k \Rightarrow\} -4x^2 + 3x + 10 - k = 0$	A1	1.1b
	$\{ "b^2 - 4ac" > 0 \Rightarrow\} 3^2 - 4(-4)(10 - k) > 0$	dM1	2.1
	$9 + 16(10 - k) > 0 \Rightarrow 169 - 16k > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		<b>(5)</b>	
<b>5 Alt 1</b>	Eliminate $y$ and forms quadratic equation $= 0$ or quadratic expression $\{= 0\}$	M1	3.1a
	$y = 2\left(\frac{1}{3}(k + 2y)\right)^2 - 5 \Rightarrow y = \frac{2}{9}(k^2 + 4ky + 4y^2) - 5$		
	$8y^2 + (8k - 9)y + 2k^2 - 45 = 0$	A1	1.1b
	$\{ "b^2 - 4ac" > 0 \Rightarrow\} (8k - 9)^2 - 4(8)(2k^2 - 45) > 0$	dM1	2.1
	$64k^2 - 144k + 81 - 64k^2 + 1440 > 0 \Rightarrow -144k + 1521 > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		<b>(5)</b>	
<b>5 Alt 2</b>	$\frac{dy}{dx} = 4x, m_t = \frac{3}{2} \Rightarrow 4x = \frac{3}{2} \Rightarrow x = \frac{3}{8}$ . So $y = 2\left(\frac{3}{8}\right)^2 - 5 = -\frac{151}{32}$	M1	3.1a
		A1	1.1b
	$k = 3\left(\frac{3}{8}\right) - 2\left(-\frac{151}{32}\right) \Rightarrow k = \dots$	dM1	2.1
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
	<b>(5)</b>		
			<b>(5 marks)</b>

Question 5 Notes:	
<b>M1:</b>	Complete strategy of eliminating $x$ or $y$ and manipulating the resulting equation to form a quadratic equation $= 0$ or a quadratic expression $\{= 0\}$
<b>A1:</b>	Correct algebra leading to either <ul style="list-style-type: none"> <li><math>-4x^2 + 3x + 10 - k = 0</math> or <math>4x^2 - 3x - 10 + k = 0</math></li> </ul> or a one-sided quadratic of either $-4x^2 + 3x + 10 - k$ or $4x^2 - 3x - 10 + k$ <ul style="list-style-type: none"> <li><math>8y^2 + (8k - 9)y + 2k^2 - 45 = 0</math></li> </ul> or a one-sided quadratic of e.g. $8y^2 + (8k - 9)y + 2k^2 - 45$
<b>dM1:</b>	Depends on the previous M mark. Interprets $3x - 2y = k$ intersecting $y = 2x^2 - 5$ at two distinct points by applying " $b^2 - 4ac > 0$ " to their quadratic equation or one-sided quadratic.
<b>B1:</b>	See scheme
<b>A1:</b>	Correct answer, e.g. <ul style="list-style-type: none"> <li><math>k &lt; \frac{169}{16}</math></li> <li><math>\left\{ k : k &lt; \frac{169}{16} \right\}</math></li> </ul>
<b>Alt 2</b>	
<b>M1:</b>	Complete strategy of using differentiation to find the values of $x$ and $y$ where $3x - 2y = k$ is a tangent to $y = 2x^2 - 5$
<b>A1:</b>	Correct algebra leading to $x = \frac{3}{8}$ , $y = -\frac{151}{32}$
<b>dM1:</b>	Depends on the previous M mark. Full method of substituting their $x = \frac{3}{8}$ , $y = -\frac{151}{32}$ into $l$ and attempting to find the value for $k$ .
<b>B1:</b>	See scheme
<b>A1:</b>	Deduces correct answer, e.g. <ul style="list-style-type: none"> <li><math>k &lt; \frac{169}{16}</math></li> <li><math>\left\{ k : k &lt; \frac{169}{16} \right\}</math></li> </ul>

Question	Scheme	Marks	AOs
<b>6(a)</b>	$f(x) = (8 - x)\ln x, x > 0$		
	Crosses $x$ -axis $\Rightarrow f(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	$x$ coordinates are 1 and 8	B1	1.1b
		(1)	
<b>(b)</b>	Complete strategy of setting $f'(x) = 0$ and rearranges to make $x = \dots$	M1	3.1a
	$\left\{ \begin{array}{l} u = (8 - x) \quad v = \ln x \\ \frac{du}{dx} = -1 \quad \frac{dv}{dx} = \frac{1}{x} \end{array} \right\}$		
	$f'(x) = -\ln x + \frac{8-x}{x}$	M1	1.1b
		A1	1.1b
	$-\ln x + \frac{8-x}{x} = 0 \Rightarrow -\ln x + \frac{8}{x} - 1 = 0$ $\Rightarrow \frac{8}{x} = 1 + \ln x \Rightarrow x = \frac{8}{1 + \ln x} *$	A1*	2.1
	(4)		
<b>(c)</b>	Evaluates both $f'(3.5)$ and $f'(3.6)$	M1	1.1b
	$f'(3.5) = 0.032951317\dots$ and $f'(3.6) = -0.058711623\dots$ Sign change and as $f'(x)$ is continuous, the $x$ coordinate of $Q$ lies between $x = 3.5$ and $x = 3.6$	A1	2.4
		(2)	
<b>(d)(i)</b>	$\{x_5 = \} 3.5340$	B1	1.1b
<b>(d)(ii)</b>	$\{x_Q = \} 3.54$ (2 dp)	B1	2.2a
		(2)	
<b>(9 marks)</b>			

Question 6 Notes:	
(a)	
<b>B1:</b>	Either <ul style="list-style-type: none"> <li>• 1 and 8</li> <li>• on Figure 2, marks 1 next to <i>A</i> and 8 next to <i>B</i></li> </ul>
(b)	
<b>M1:</b>	Recognises that <i>Q</i> is a stationary point (and not a root) and applies a complete strategy of setting $f'(x) = 0$ and rearranges to make $x = \dots$
<b>M1:</b>	Applies $vu' + uv'$ , where $u = 8 - x$ , $v = \ln x$
	<b>Note:</b> This mark can be recovered for work in part (c)
<b>A1:</b>	$(8 - x)\ln x \rightarrow -\ln x + \frac{8-x}{x}$ , or equivalent
	<b>Note:</b> This mark can be recovered for work in part (c)
<b>A1*:</b>	Correct proof with no errors seen in working.
(c)	
<b>M1:</b>	Evaluates both $f'(3.5)$ and $f'(3.6)$
<b>A1:</b>	$f'(3.5) = \text{awrt } 0.03$ and $f'(3.6) = \text{awrt } -0.06$ or $f'(3.6) = -0.05$ (truncated) <b>and</b> a correct conclusion
(d)(i)	
<b>B1:</b>	See scheme
(d)(ii)	
<b>B1:</b>	Deduces (e.g. by the use of further iterations) that the $x$ coordinate of <i>Q</i> is 3.54 accurate to 2 dp <b>Note:</b> $3.5 \rightarrow 3.55119 \rightarrow 3.52845 \rightarrow 3.53848 \rightarrow 3.53404 \rightarrow 3.53600 \rightarrow 3.53514$ ( $\rightarrow 3.535518\dots$ )

Question	Scheme	Marks	AOs
<b>7(a)</b>	$\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$	B1	3.3
	$\int \frac{1}{p} dp = \int k dt$	M1	1.1b
	$\ln p = kt \{+ c\}$	A1	1.1b
	$\ln p = kt + c \Rightarrow p = e^{kt+c} = e^{kt} e^c \Rightarrow p = ae^{kt} *$	A1 *	2.1
		<b>(4)</b>	
<b>(b)</b>	$p = ae^{kt} \Rightarrow \ln p = \ln a + kt$ and evidence of understanding that either <ul style="list-style-type: none"> <li>• gradient = <math>k</math> or "<math>M</math>" = <math>k</math></li> <li>• vertical intercept = <math>\ln a</math> or "<math>C</math>" = <math>\ln a</math></li> </ul>	M1	2.1
	gradient = $k = 0.14$	A1	1.1b
	vertical intercept = $\ln a = 3.95 \Rightarrow a = e^{3.95} = 51.935 = 52$ (2 sf)	A1	1.1b
		<b>(3)</b>	
<b>(c)</b>	e.g. <ul style="list-style-type: none"> <li>• <math>p = ae^{kt} \Rightarrow p = a(e^k)^t = ab^t</math>,</li> <li>• <math>p = 52e^{0.14t} \Rightarrow p = 52(e^{0.14})^t</math></li> </ul>	B1	2.2a
	$b = 1.15$ which can be implied by $p = 52(1.15)^t$	B1	1.1b
		<b>(2)</b>	
<b>(d)(i)</b>	Initial area (i.e. "52" mm <sup>2</sup> ) of bacterial culture that was first placed onto the circular dish.	B1	3.4
<b>(d)(ii)</b>	E.g. <ul style="list-style-type: none"> <li>• Rate of increase per hour of the area of bacterial culture</li> <li>• The area of bacterial culture increases by "15%" each hour</li> </ul>	B1	3.4
		<b>(2)</b>	
<b>(e)</b>	The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area.	B1	3.5b
		<b>(1)</b>	
<b>(12 marks)</b>			

Question 7 Notes:	
(a)	
<b>B1:</b>	Translates the scientist's statement regarding proportionality into a differential equation, which involves a constant of proportionality. e.g. $\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$
<b>M1:</b>	Correct method of separating the variables $p$ and $t$ in their differential equation
<b>A1:</b>	$\ln p = kt$ , with or without a constant of integration
<b>A1*:</b>	Correct proof with no errors seen in working.
(b)	
<b>M1:</b>	See scheme
<b>A1:</b>	Correctly finds $k = 0.14$
<b>A1:</b>	Correctly finds $a = 52$
(c)	
<b>B1:</b>	Uses algebra to correctly deduce either <ul style="list-style-type: none"> <li>• <math>p = ab^t</math> from <math>p = ae^{kt}</math></li> <li>• <math>p = "52"(e^{0.14})^t</math> from <math>p = "52"e^{0.14t}</math></li> </ul>
<b>B1:</b>	See scheme
(d)(i)	
<b>B1:</b>	See scheme
(d)(ii)	
<b>B1:</b>	See scheme
(e)	
<b>B1:</b>	Gives a correct long-term limitation of the model for $p$ . (See scheme).

Question	Scheme	Marks	AOs
<b>8(a)</b>	$\frac{dV}{dt} = 160\pi, V = \frac{1}{3}\pi h^2(75 - h) = 25\pi h^2 - \frac{1}{3}\pi h^3$		
	$\frac{dV}{dh} = 50\pi h - \pi h^2$	M1	1.1b
		A1	1.1b
	$\left\{ \frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \right\} (50\pi h - \pi h^2) \frac{dh}{dt} = 160\pi$	M1	3.1a
	When $h = 10$ , $\left\{ \frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow \right\} \frac{160\pi}{50\pi(10) - \pi(10)^2} \left\{ = \frac{160\pi}{400\pi} \right\}$	dM1	3.4
	$\frac{dh}{dt} = 0.4 \text{ (cms}^{-1}\text{)}$	A1	1.1b
	<b>(5)</b>		
<b>(b)</b>	$\frac{dh}{dt} = \frac{300\pi}{50\pi(20) - \pi(20)^2}$	M1	3.4
	$\frac{dh}{dt} = 0.5 \text{ (cms}^{-1}\text{)}$	A1	1.1b
		<b>(2)</b>	
<b>(7 marks)</b>			

**Question 8 Notes:**

<b>(a)</b>	
<b>M1:</b>	Differentiates $V$ with respect to $h$ to give $\pm \alpha h \pm \beta h^2, \alpha \neq 0, \beta \neq 0$
<b>A1:</b>	$50\pi h - \pi h^2$
<b>M1:</b>	Attempts to solve the problem by applying a complete method of $\left( \text{their } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 160\pi$
<b>M1:</b>	Depends on the previous M mark. Substitutes $h = 10$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{160\pi}{\left( \text{their } \frac{dV}{dh} \right)}$
<b>A1:</b>	Obtains the correct answer 0.4
<b>(b)</b>	
<b>M1:</b>	Realises that rate for of $160\pi \text{ cm}^3 \text{ s}^{-1}$ for $0 \leq h \leq 12$ has no effect when the rate is increased to $300\pi \text{ cm}^3 \text{ s}^{-1}$ for $12 < h \leq 24$ and so substitutes $h = 20$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{300\pi}{\left( \text{their } \frac{dV}{dh} \right)}$
<b>A1:</b>	Obtains the correct answer 0.5

Question	Scheme	Marks	AOs
<b>9(a)</b>	E.g. midpoint $PQ = \left( \frac{-9 + 15}{2}, \frac{8 - 10}{2} \right)$	M1	1.1b
	$= (3, -1)$ , which is the centre point A, so $PQ$ is the diameter of the circle.	A1	2.1
		(2)	
<b>(a)</b> <b>Alt 1</b>	$m_{PQ} = \frac{-10 - 8}{15 - -9} = -\frac{3}{4} \Rightarrow PQ: y - 8 = -\frac{3}{4}(x - -9)$	M1	1.1b
	$PQ: y = -\frac{3}{4}x + \frac{5}{4}$ . So $x = 3 \Rightarrow y = -\frac{3}{4}(3) + \frac{5}{4} = -1$ so $PQ$ is the diameter of the circle.	A1	2.1
		(2)	
<b>(a)</b> <b>Alt 2</b>	$PQ = \sqrt{(-9 - 15)^2 + (8 - -10)^2} \{ = \sqrt{900} = 30 \}$ <b>and either</b>	M1	1.1b
	<ul style="list-style-type: none"> <li><math>AP = \sqrt{(3 - -9)^2 + (-1 - 8)^2} \{ = \sqrt{225} = 15 \}</math></li> <li><math>AQ = \sqrt{(3 - 15)^2 + (-1 - -10)^2} \{ = \sqrt{225} = 15 \}</math></li> </ul>		
	e.g. as $PQ = 2AP$ , then $PQ$ is the diameter of the circle.	A1	2.1
		(2)	
<b>(b)</b>	Uses Pythagoras in a correct method to find either the radius or diameter of the circle.	M1	1.1b
	$(x - 3)^2 + (y + 1)^2 = 225$ (or $(15)^2$ )	M1	1.1b
		A1	1.1b
		(3)	
<b>(c)</b>	Distance $= \sqrt{("15")^2 - (10)^2}$ or $= \frac{1}{2}\sqrt{(2("15"))^2 - (2(10))^2}$	M1	3.1a
	$\{ = \sqrt{125} \} = 5\sqrt{5}$	A1	1.1b
		(2)	
<b>(d)</b>	$\sin(\hat{A}RQ) = \frac{20}{2("15")}$ or $\hat{A}RQ = 90 - \cos^{-1}\left(\frac{10}{"15"}\right)$	M1	3.1a
	$\hat{A}RQ = 41.8103... = 41.8^\circ$ (to 0.1 of a degree)	A1	1.1b
		(2)	
<b>(9 marks)</b>			

Question 9 Notes:	
(a)	
<b>M1:</b>	Uses a correct method to find the midpoint of the line segment $PQ$
<b>A1:</b>	Completes proof by obtaining $(3, -1)$ and gives a correct conclusion.
(a)	
<b>Alt 1</b>	
<b>M1:</b>	Full attempt to find the equation of the line $PQ$
<b>A1:</b>	Completes proof by showing that $(3, -1)$ lies on $PQ$ and gives a correct conclusion.
(a)	
<b>Alt 2</b>	
<b>M1:</b>	Attempts to find distance $PQ$ and either one of distance $AP$ or distance $AQ$
<b>A1:</b>	Correctly shows either <ul style="list-style-type: none"> <li><math>PQ = 2AP</math>, supported by <math>PQ = 30</math>, <math>AP = 15</math> and gives a correct conclusion</li> <li><math>PQ = 2AQ</math>, supported by <math>PQ = 30</math>, <math>AQ = 15</math> and gives a correct conclusion</li> </ul>
(b)	
<b>M1:</b>	<b>Either</b> <ul style="list-style-type: none"> <li>uses Pythagoras correctly in order to find the <b>radius</b>. Must clearly be identified as the <b>radius</b>. E.g. <math>r^2 = (-9 - 3)^2 + (8 + 1)^2</math> or <math>r = \sqrt{(-9 - 3)^2 + (8 + 1)^2}</math> or <math>r^2 = (15 - 3)^2 + (-10 + 1)^2</math> or <math>r = \sqrt{(15 - 3)^2 + (-10 + 1)^2}</math></li> </ul> <b>or</b> <ul style="list-style-type: none"> <li>uses Pythagoras correctly in order to find the <b>diameter</b>. Must clearly be identified as the <b>diameter</b>. E.g. <math>d^2 = (15 + 9)^2 + (-10 - 8)^2</math> or <math>d = \sqrt{(15 + 9)^2 + (-10 - 8)^2}</math></li> </ul> <b>Note:</b> This mark can be implied by just 30 clearly seen as the <b>diameter</b> or 15 clearly seen as the <b>radius</b> (may be seen or implied in their circle equation)
<b>M1:</b>	Writes down a circle equation in the form $(x \pm "3")^2 + (y \pm "-1")^2 = (\text{their } r)^2$
<b>A1:</b>	$(x - 3)^2 + (y + 1)^2 = 225$ or $(x - 3)^2 + (y + 1)^2 = 15^2$ or $x^2 - 6x + y^2 + 2y - 215 = 0$
(c)	
<b>M1:</b>	Attempts to solve the problem by using the circle property "the perpendicular from the centre to a chord bisects the chord" and so applies Pythagoras to write down an expression of the form $\sqrt{(\text{their "15"})^2 - (10)^2}$ .
<b>A1:</b>	$5\sqrt{5}$ by correct solution only
(d)	
<b>M1:</b>	Attempts to solve the problem by e.g. using the circle property "the angle in a semi-circle is a right angle" and writes down either $\sin(\hat{A}RQ) = \frac{20}{2(\text{their "15"})}$ or $\hat{A}RQ = 90 - \cos^{-1}\left(\frac{10}{\text{their "15"}}\right)$
<b>Note:</b>	Also allow $\cos(\hat{A}RQ) = \frac{15^2 + (2(5\sqrt{5}))^2 - 15^2}{2(15)(2(5\sqrt{5}))} \left\{ = \frac{\sqrt{5}}{3} \right\}$
<b>A1:</b>	41.8 by correct solution only

Question	Scheme	Marks	AOs
<b>10 (a)</b>	$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a
		(1)	
<b>(b)</b>	Attempts to apply $\int y \frac{dx}{dt} dt$	M1	3.1a
	$\left\{ \int y \frac{dx}{dt} dt = \right\} = \int \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right) dt$	A1	1.1b
	$\frac{1}{(t+1)(t+2)} \equiv \frac{A}{t+1} + \frac{B}{t+2} \Rightarrow 1 \equiv A(t+2) + B(t+1)$	M1	3.1a
	$\{A = 1, B = -1 \Rightarrow \}$ gives $\frac{1}{t+1} - \frac{1}{t+2}$	A1	1.1b
	$\left\{ \int \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt = \right\} \ln(t+1) - \ln(t+2)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln(t+1) - \ln(t+2)]_0^2 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right) *$	A1*	2.1
		(8)	
<b>(b)</b> <b>Alt 1</b>	Attempts to apply $\int y dx = \int \frac{1}{e^x - 2 + 1} dx = \int \frac{1}{e^x - 1} dx,$ with a substitution of $u = e^x - 1$	M1	3.1a
	$\left\{ \int y dx \right\} = \int \left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right) du$	A1	1.1b
	$\frac{1}{u(u+1)} \equiv \frac{A}{u} + \frac{B}{u+1} \Rightarrow 1 \equiv A(u+1) + Bu$	M1	3.1a
	$\{A = 1, B = -1 \Rightarrow \}$ gives $\frac{1}{u} - \frac{1}{u+1}$	A1	1.1b
	$\left\{ \int \left(\frac{1}{u} - \frac{1}{u+1}\right) du = \right\} \ln u - \ln(u+1)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln u - \ln(u+1)]_1^3 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right) *$	A1 *	2.1
		(8)	
<b>(9 marks)</b>			

Question	Scheme	Marks	AOs
<b>10 (b)</b> <b>Alt 2</b>	Attempts to apply $\int ydx = \int \frac{1}{e^x - 2 + 1} dx = \int \frac{1}{e^x - 1} dx$ , with a substitution of $v = e^x$	M1	3.1a
	$\left\{ \int ydx \right\} = \int \left( \frac{1}{v-1} \right) \left( \frac{1}{v} \right) dv$	A1	1.1b
	$\frac{1}{(v-1)v} \equiv \frac{A}{v-1} + \frac{B}{v} \Rightarrow 1 \equiv Av + B(v-1)$	M1	3.1a
	$\{A = 1, B = -1 \Rightarrow \}$ gives $\frac{1}{v-1} - \frac{1}{v}$	A1	1.1b
	$\left\{ \int \left( \frac{1}{v-1} - \frac{1}{v} \right) dv = \right\} \ln(v-1) - \ln v$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln(v-1) - \ln v]_2^4 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln \left( \frac{(3)(2)}{4} \right) = \ln \left( \frac{6}{4} \right)$		
	$= \ln \left( \frac{3}{2} \right) *$	A1 *	2.1
	<b>(8)</b>		

Question 10 Notes:	
(a)	
<b>B1:</b>	Uses $x = \ln(t + 2)$ with $t > -\frac{2}{3}$ to deduce the correct domain, $x > \ln\left(\frac{4}{3}\right)$
(b)	
<b>M1:</b>	Attempts to solve the problem by either <ul style="list-style-type: none"> <li>• a parametric process or</li> <li>• a Cartesian process with a substitution of either <math>u = e^x - 1</math> or <math>v = e^x</math></li> </ul>
<b>A1:</b>	Obtains <ul style="list-style-type: none"> <li>• <math>\int \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right) dt</math> from a parametric approach</li> <li>• <math>\int \left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right) du</math> from a Cartesian approach with <math>u = e^x - 1</math></li> <li>• <math>\int \left(\frac{1}{v-1}\right)\left(\frac{1}{v}\right) dv</math> from a Cartesian approach with <math>v = e^x</math></li> </ul>
<b>M1:</b>	Applies a strategy of attempting to express either $\frac{1}{(t+1)(t+2)}$ , $\frac{1}{u(u+1)}$ or $\frac{1}{(v-1)v}$ as partial fractions
<b>A1:</b>	Correct partial fractions for their method
<b>M1:</b>	Integrates to give either <ul style="list-style-type: none"> <li>• <math>\pm\alpha \ln(t+1) \pm \beta \ln(t+2)</math></li> <li>• <math>\pm\alpha \ln u \pm \beta \ln(u+1)</math>; <math>\alpha, \beta \neq 0</math>, where <math>u = e^x - 1</math></li> <li>• <math>\pm\alpha \ln(v-1) \pm \beta \ln v</math>; <math>\alpha, \beta \neq 0</math>, where <math>v = e^x</math></li> </ul>
<b>A1:</b>	Correct integration for their method
<b>M1:</b>	Either <ul style="list-style-type: none"> <li>• Parametric approach: Deduces and applies limits of 2 and 0 in <math>t</math> and subtracts the correct way round</li> <li>• Cartesian approach: Deduces and applies limits of 3 and 1 in <math>u</math>, where <math>u = e^x - 1</math>, and subtracts the correct way round</li> <li>• Cartesian approach: Deduces and applies limits of 4 and 2 in <math>v</math>, where <math>v = e^x</math>, and subtracts the correct way round</li> </ul>
<b>A1*:</b>	Correctly shows that the area of $R$ is $\ln\left(\frac{3}{2}\right)$ , with no errors seen in their working

Question	Scheme	Marks	AOs
<b>11</b>	Arithmetic sequence, $T_2 = 2k$ , $T_3 = 5k - 10$ , $T_4 = 7k - 14$		
	$(5k - 10) - (2k) = (7k - 14) - (5k - 10) \Rightarrow k = \dots$	M1	2.1
	$\{3k - 10 = 2k - 4 \Rightarrow\} \quad k = 6$	A1	1.1b
	$\{k = 6 \Rightarrow\} \quad T_2 = 12, T_3 = 20, T_4 = 28$ . So $d = 8, a = 4$	M1	2.2a
	$S_n = \frac{n}{2}(2(4) + (n-1)(8))$	M1	1.1b
	$= \frac{n}{2}(8 + 8n - 8) = 4n^2 = (2n)^2$ which is a square number	A1	2.1
		<b>(5)</b>	
<b>(5 marks)</b>			
<b>Question 11 Notes:</b>			
<b>M1:</b>	Complete method to find the value of $k$		
<b>A1:</b>	Uses a correct method to find $k = 6$		
<b>M1:</b>	Uses their value of $k$ to deduce the common difference and the first term ( $\neq T_2$ ) of the arithmetic series.		
<b>M1:</b>	Applies $S_n = \frac{n}{2}(2a + (n-1)d)$ with their $a \neq T_2$ and their $d$ .		
<b>A1:</b>	Correctly shows that the sum of the series is $(2n)^2$ and makes an appropriate conclusion.		

Question	Scheme	Marks	AOs
<b>12</b>	Complete process to find at least one set of coordinates for $P$ . The process must include evidence of <ul style="list-style-type: none"> <li>differentiating</li> <li>setting <math>\frac{dy}{dx} = 0</math> to find <math>x = \dots</math></li> <li>substituting <math>x = \dots</math> into <math>\sin x + \cos y = 0.5</math> to find <math>y = \dots</math></li> </ul>	M1	3.1a
	<del>cos</del> $\left. \begin{matrix} \cos x - \sin y \frac{dy}{dx} = 0 \end{matrix} \right\}$	B1	1.1b
	Applies $\frac{dy}{dx} = 0$ (e.g. $\cos x = 0$ or $\frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0 \Rightarrow x = \dots$ )	M1	2.2a
	giving at least one of either $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$	A1	1.1b
	$x = \frac{\pi}{2} \Rightarrow \sin\left(\frac{\pi}{2}\right) + \cos y = 0.5 \Rightarrow \cos y = -\frac{1}{2} \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	M1	1.1b
	So in specified range, $(x, y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ , by cso	A1	1.1b
	$x = -\frac{\pi}{2} \Rightarrow \sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5 \Rightarrow \cos y = 1.5$ has no solutions, and so there are exactly 2 possible points $P$ .	B1	2.1
		<b>(7)</b>	

**(7 marks)**

**Question 12 Notes:**

<b>M1:</b>	See scheme
<b>B1:</b>	Correct differentiated equation. E.g. $\cos x - \sin y \frac{dy}{dx} = 0$
<b>M1:</b>	Uses the information “the tangent to $C$ at the point $P$ is parallel to the $x$ -axis” to deduce and apply $\frac{dy}{dx} = 0$ and finds $x = \dots$
<b>A1:</b>	See scheme
<b>M1:</b>	For substituting one of their values from $\frac{dy}{dx} = 0$ into $\sin x + \cos y = 0.5$ and so finds $x = \dots, y = \dots$
<b>A1:</b>	Selects coordinates for $P$ on $C$ satisfying $\frac{dy}{dx} = 0$ and $-\frac{\pi}{2}, x < \frac{3\pi}{2}, -\pi < y < \pi$ i.e. finds $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ and no other points by correct solution only
<b>B1:</b>	Complete argument to show that there are exactly 2 possible points $P$ .

Question	Scheme	Marks	AOs
<b>13(a)</b>	$\operatorname{cosec}2x + \cot 2x \equiv \cot x, \quad x \neq 90n^\circ, \quad n \in \mathbb{Z}$		
	$\operatorname{cosec}2x + \cot 2x = \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$	M1	1.2
	$= \frac{1 + \cos 2x}{\sin 2x}$	M1	1.1b
	$= \frac{1 + 2\cos^2 x - 1}{2\sin x \cos x} = \frac{2\cos^2 x}{2\sin x \cos x}$	M1	2.1
	$= \frac{\cos x}{\sin x} = \cot x \quad *$	A1*	2.1
		<b>(5)</b>	
<b>(b)</b>	$\operatorname{cosec}(4\theta + 10^\circ) + \cot(4\theta + 10^\circ) = \sqrt{3}; \quad 0^\circ < \theta < 180^\circ,$		
	$\cot(2\theta \pm \dots^\circ) = \sqrt{3}$	M1	2.2a
	$2\theta \pm \dots = 30^\circ \Rightarrow \theta = 12.5^\circ$	M1	1.1b
		A1	1.1b
	$2\theta + 5^\circ = 180^\circ + PV^\circ \Rightarrow \theta = \dots^\circ$	M1	2.1
	$\theta = 102.5^\circ$	A1	1.1b
	<b>(5)</b>		
<b>(10 marks)</b>			

Question 13 Notes:	
(a)	
<b>M1:</b>	Writes $\operatorname{cosec}2x = \frac{1}{\sin 2x}$ and $\cot 2x = \frac{\cos 2x}{\sin 2x}$
<b>M1:</b>	Combines into a single fraction with a common denominator
<b>M1:</b>	Applies $\sin 2x = 2\sin x \cos x$ to the denominator <b>and</b> applies either <ul style="list-style-type: none"> <li>• <math>\cos 2x = 2\cos^2 x - 1</math></li> <li>• <math>\cos 2x = 1 - 2\sin^2 x</math> and <math>\sin^2 x + \cos^2 x = 1</math></li> <li>• <math>\cos 2x = \cos^2 x - \sin^2 x</math> and <math>\sin^2 x + \cos^2 x = 1</math></li> </ul> to the numerator and manipulates to give a one term numerator expression
<b>A1:</b>	Correct algebra leading to $\frac{2\cos^2 x}{2\sin x \cos x}$ or equivalent.
<b>A1*:</b>	Correct proof with correct notation and no errors seen in working
(b)	
<b>M1:</b>	Uses the result in part (a) in an attempt to deduce either $2x = 4\theta + 10$ or $x = 2\theta + \dots$ and uses $x = 2\theta + \dots$ to write down or imply $\cot(2\theta \pm \dots) = \sqrt{3}$
<b>M1:</b>	Applies $\operatorname{arccot}(\sqrt{3}) = 30^\circ$ or $\arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$ and attempts to solve $2\theta \pm \dots = 30^\circ$ to give $\theta = \dots$
<b>A1:</b>	Uses a correct method to obtain $\theta = 12.5^\circ$
<b>M1:</b>	Uses $2\theta + 5 = 180 +$ their $PV^\circ$ in a complete method to find the second solution, $\theta = \dots$
<b>A1:</b>	Uses a correct method to obtain $\theta = 102.5^\circ$ , with no extra solutions given either inside or outside the required range $0, \theta < 180^\circ$

Question	Scheme	Marks	AOs
<b>14 (i)</b>	For an explanation or statement to show when the claim $3^x \dots 2^x$ fails This could be e.g. <ul style="list-style-type: none"> <li>when <math>x = -1</math>, <math>\frac{1}{3} &lt; \frac{1}{2}</math> or <math>\frac{1}{3}</math> is not greater than or equal to <math>\frac{1}{2}</math></li> <li>when <math>x &lt; 0</math>, <math>3^x &lt; 2^x</math> or <math>3^x</math> is not greater than or equal to <math>2^x</math></li> </ul>	M1	2.3
	followed by an explanation or statement to show when the claim $3^x \dots 2^x$ is true. This could be e.g. <ul style="list-style-type: none"> <li><math>x = 2</math>, <math>9 \dots 4</math> or 9 is greater than or equal to 4</li> <li>when <math>x \dots 0</math>, <math>3^x \dots 2^x</math></li> </ul> and a correct conclusion. E.g. <ul style="list-style-type: none"> <li>so the claim <math>3^x \dots 2^x</math> is sometimes true</li> </ul>	A1	2.4
		<b>(2)</b>	
<b>(ii)</b>	Assume that $\sqrt{3}$ is a rational number So $\sqrt{3} = \frac{p}{q}$ , where $p$ and $q$ integers, $q \neq 0$ , and the HCF of $p$ and $q$ is 1	M1	2.1
	$\Rightarrow p = \sqrt{3}q \Rightarrow p^2 = 3q^2$	M1	1.1b
	$\Rightarrow p^2$ is divisible by 3 and so $p$ is divisible by 3	A1	2.2a
	So $p = 3c$ , where $c$ is an integer From earlier, $p^2 = 3q^2 \Rightarrow (3c)^2 = 3q^2$	M1	2.1
	$\Rightarrow q^2 = 3c^2 \Rightarrow q^2$ is divisible by 3 and so $q$ is divisible by 3	A1	1.1b
	As both $p$ and $q$ are both divisible by 3 then the HCF of $p$ and $q$ is not 1 This contradiction implies that $\sqrt{3}$ is an irrational number	A1	2.4
		<b>(6)</b>	
<b>(8 marks)</b>			

<b>Question 14 Notes:</b>	
<b>(i)</b>	
<b>M1:</b>	See scheme
<b>A1:</b>	See scheme
<b>(ii)</b>	
<b>M1:</b>	Uses a method of proof by contradiction by initially assuming that $\sqrt{3}$ is rational and expresses $\sqrt{3}$ in the form $\frac{p}{q}$ , where $p$ and $q$ are correctly defined.
<b>M1:</b>	Writes $\sqrt{3} = \frac{p}{q}$ and rearranges to make $p^2$ the subject
<b>A1:</b>	Uses a logical argument to prove that $p$ is divisible by 3
<b>M1:</b>	Uses the result that $p$ is divisible by 3, (to construct the initial stage of proving that $q$ is also divisible by 3), by substituting $p = 3c$ into their expression for $p^2$
<b>A1:</b>	Hence uses a correct argument, in the same way as before, to deduce that $q$ is also divisible by 3
<b>A1:</b>	Completes the argument (as detailed on the scheme) that $\sqrt{3}$ is irrational.
	<b>Note:</b> All the previous 5 marks need to be scored in order to obtain the final A mark.